$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(1+h)-f(1)}{h} = \frac{h+4h^2-5h^3}{h}$$

$$\lim_{h\to 0} 1+4h-5h^2 = 1$$

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 0$$

5. $\frac{1}{x^3} = x^{-3-1}$

$-3x^{-4}$

8. $\sqrt[5]{x} = x^{\frac{1}{5}}$

$\frac{1}{5}x^{-\frac{4}{5}} = \frac{1}{5\sqrt[5]{x^4}}$

16. $(x+2)^2$

$x^2 + 4x + 4$

$2x + 4$

18. $x^2 - \frac{4}{x^3}$

$f(x) = x^2 - 4x^{-3}$

$f'(x) = 2x + 12x^{-4}$

19. $x + \frac{1}{x^2}$

$x + x^{-2}$

$\frac{dy}{dx} = 1 - 2x^{-3}$

21.

$f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = \frac{x^3}{x^2} - \frac{3\cancel{x}^{2}}{\cancel{x}^2} + \frac{4}{x^2}$

$x^{-2}(x^3 - 3x^2 + 4)$

$x - 3 + 4x^{-2}$

0

$f'(x) = 1 - 8x^{-3}$

27. $f(x) = 3 - \frac{3}{5x}$ $\quad -\frac{3}{5}x^{-1}$

$\frac{3}{5}x^{-2}$

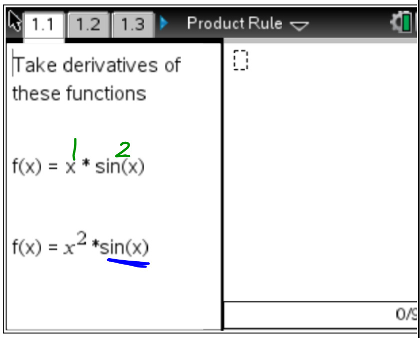
$\frac{3}{5}\left(\frac{3}{5}\right)^{-2}$

$\cancel{\frac{3}{5}}\left(\frac{5}{3}\right)^{2} = \frac{5}{3}$

$\cancel{\frac{3}{5}} \cdot \cancel{\frac{5}{3}} \cdot \frac{5}{3}$

3.3b Rules for Differentiation

Use Product Rule to discover the rule for taking derivatives of products of functions



1 d2 + 2 d1

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(x^2+1)(x^3-2) \qquad x^5 - 2x^2 + x^3 - 2$$

$$(x^2+1)(3x^2) + (x^3-2)(2x)$$

$$(x^2)(x^3) = x^5$$

$$5x^4$$

$$\cancel{2x \cdot 3x^2}$$

$$\cancel{6x^3}$$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) =$$

Differentiate: $= (x^2+1)^1(3x-2)^{-1}$

$$y' = (x^2+1)\left(-(3x-2)^{-2}(3)\right) + (3x-2)^{-1}(2x)$$

$$\frac{-3(x^2+1)}{(3x-2)^2} + \frac{2x\,(3x-2)}{(3x-2)^2}$$

$$y = \frac{\overset{\text{high}}{x^2+1}}{\underset{\text{low}}{3x-2}}$$

$$y' = \frac{\overset{\text{d high}}{2x}\,\overset{\text{low}}{(3x-2)} - \overset{\text{d low}}{3}\,\overset{\text{high}}{(x^2+1)}}{(3x-2)^2}$$

$$(\text{low})^2$$

$$\text{quotient rule} = \frac{\text{low d high} - \text{high d low}}{(\text{low})^2}$$

Find an equation for the line tangent to the curve

$$y = \frac{x^2+3}{2x} \quad \text{at } x=1$$

$(1, 2)$

$$y' = \left.\frac{2x(2x) - (x^2+3)(2)}{(2x)^2}\right|_{x=1}$$

$$= \frac{2(2)-(4)(2)}{4} = \frac{-4}{4} = -1$$

$$y = -1(x-1)+2$$

$$\lim_{h\to 0} \frac{\dfrac{(1+h)^2+3}{2(1+h)} - \left(\dfrac{1^2+3}{2}\right)}{h}$$

Higher order derivatives

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\frac{dy}{dx} = \frac{d^2y}{dx^2}$$

$$y''' = \frac{dy''}{dx} = \frac{d}{dx}\frac{dy}{dx}\frac{dy}{dx} = \frac{d^3y}{dx^3}$$