7. $f(x) = \sqrt{x+1}$ @ $a=3$

$$\lim_{x\to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$$

$$\lim_{x\to 3} \frac{(\sqrt{x+1} - 2)}{x-3} \frac{(\sqrt{x+1} + 2)}{(\sqrt{x+1} + 2)}$$

$$\lim_{x\to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1} + 2)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{4}$$

5. $f(x) = \frac{1}{x}$ @ $a=2$

$$\lim_{x\to 2} \frac{\frac{2\cdot 1}{2\cdot x} - \frac{1x}{2x}}{x-2} = \frac{\frac{2-x}{2x}}{\cancel{x-2}} \cdot \frac{1}{x-2}$$

$$= \frac{-(\cancel{-2+x})}{2x} \cdot \frac{1}{\cancel{x-2}}$$

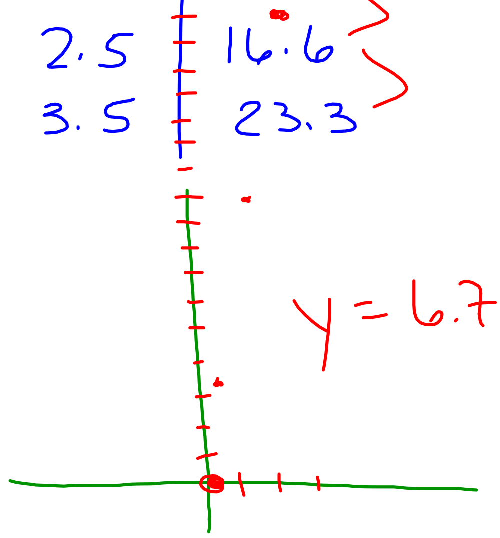
$$\lim_{x\to 2} \frac{-1}{2x} = -\frac{1}{4}$$

time sec.	ft	
0	0	
1	3.3	> 3.3
2	13.3	> 10
3	29.9	>
4	53.2	

1 < 1 < 1 <

time	ft. / sec.
0	0
.5	3.3
1.5	10
2.5	16.6
3.5	23.3

ft / sec



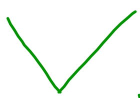
$y = 6.7x$

3.2 Differentiability - can you find the derivative?

A function will not have a derivative at a point $(a, f(a))$ if the slopes of the secant lines fail to approach a limit as x approaches a.

Let's investigate several functions
(using differentiability)

corner $y = |x| + 3$ no tangent line →



no der. because left slope ≠ right slope

cusp $y = x^{\frac{2}{3}}$ no tangent line

no der. same reason

vertical tangent $y = \sqrt[3]{x}$

no der. because slope is und.

discontinuity

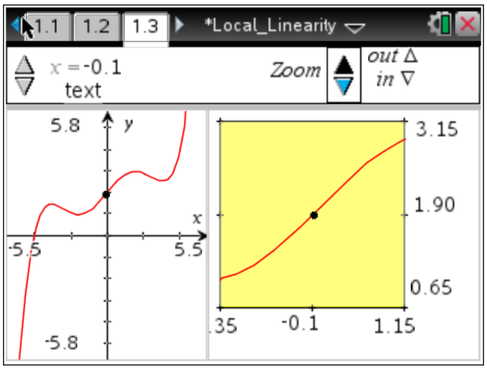
$$y = \begin{cases} 1, x > 0 \\ -1, x < 0 \end{cases}$$

no der. because not continuous

Local Linearity

A good way to think of differentiable functions is that they are locally linear.

Use Local_Linearity.tns to explore and write a definition of "locally linear"



Derivatives on your calculator:

$3x^2+5x \Big|_{x=2}$

numeric, numerical derivative

not a slope function

but an actual slope value

nDeriv(3X²+5X,X,2)

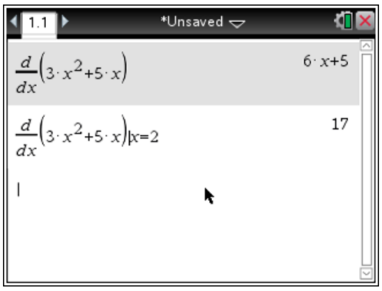
17

$\frac{d}{dx}(3X^2+5X)\big|_{X=2}$

17

symbolic → slope function

(CAS)



Differentiability implies continuity

if a function has a der.
the function is cont.

is the converse true?

not if the function is cont.
true then the function has a der.