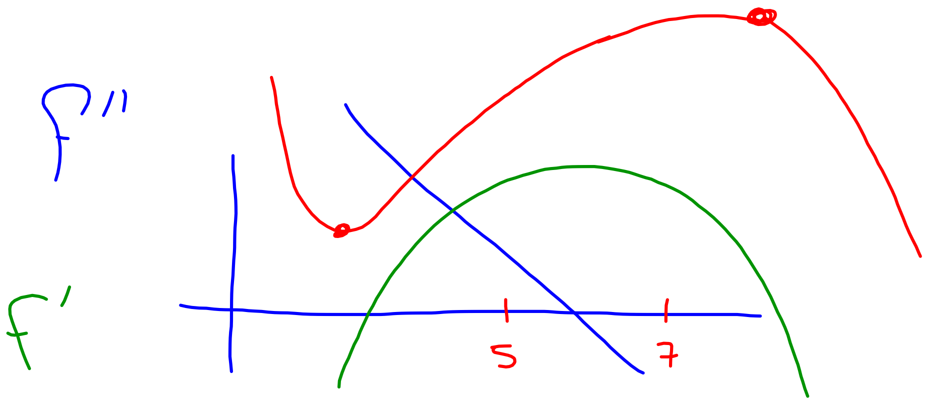


9.

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = t^2+t-6=0$$

$$(t+3)(t-2)=0$$

$$t=2,-3$$

Optimization:

1.

$x \cdot y = 192$ $\quad x = \boxed{\frac{192}{y}}$

$S = x + 3y$

$x = \frac{192}{8}$

24

$S = \frac{192}{y} + 3y$

$\frac{dS}{dy} = -\frac{192}{y^2} + 3 = 0$

$f' = 0$

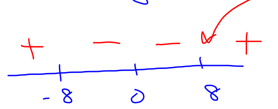
$+\frac{192}{y^2} = +3$

$\frac{y^2}{192} = \frac{1}{3}$

$y^2 = 64$

$y = \cancel{\pm} 8$

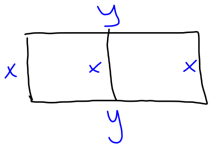
$f' = DNE$ $\quad y = 0$



5.

take der. $* A = xy$

$* \; 3x + 2y = 102$



$y = \frac{102}{2} - \frac{3x}{2}$

$A = x\left(51 - \frac{3}{2}x\right)$

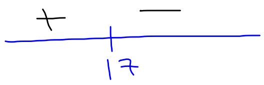
$51 - \frac{51}{2}$

$y = \frac{51}{2}$

$A = 51x - \frac{3}{2}x^2$

$A' = 51 - 3x = 0$

$x = 17$



max

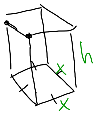
max @ $x = 17$

$y = \frac{51}{2}$

$A = \frac{51}{2} \cdot 17 = \frac{867}{2}$

Back

A square prism w/ an open top has a V of 102 cm³. What are the dimensions needed to minimize the SA?



$V = x^2h = 102$ $\quad h = \frac{102}{x^2}$

$SA = 4xh + x^2$

$SA = 4x\left(\frac{102}{x^2}\right) + x^2$

$SA = \frac{408}{x} + x^2$

$SA' = -\frac{408}{x^2} + 2x = 0$

$\frac{-408 + 2x^3}{x^2} = 0$

$x = 0$ $\quad 2x^3 = 408$

$x^3 = 204$

$x = 5.887$

$$\begin{array}{ccccc} & - & & + & \\ \hline 0 & & 5.887 & & \end{array}$$

$x = 5.887$

$h = 2.943$

Series:

what values of x . . . series converge?

Ratio $\quad |\ \ | < 1$

must check endpts.

Geom. $\quad |r| < 1$

never include endpts.

Error:

$$\left| f(x) - P(x) \right|$$

E

Lagrange
Taylor Remainder

$$E < \frac{M(x-a)^{n+1}}{(n+1)!}$$

max value on
n+1 derivative
w/in interval

Alternating

$$E < \left| \begin{matrix}\text{next}\\ \text{term}\end{matrix} \right|$$

Actual
Error

$$E = \left| f(a) - P(a) \right|$$

---

n^th term

$$\sum_{n=2}^{\infty} \boxed{\frac{2n+1}{3n-5}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{p}}$$

$p > 1$ conv.

$p \leq 1$ div.

$$\begin{tikzcd} \text{if div.} \arrow[r, bend left] & \text{then div.} \\ a_n < b_n & \\ \text{con.} & \text{if con.} \arrow[l, bend left, "\text{then}"] \end{tikzcd}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

$$\lim_{n\to\infty} \frac{\frac{3n^2+5}{5n^4+1} \cdot \frac{n^2}{1}}{\frac{1}{\cancel{n^2}}} = \frac{3}{5}$$

$\sin x$

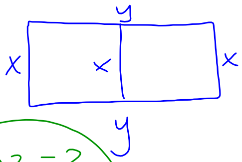
$\cos x$

e^x

$\ln x$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3$$

Optimization:



$3x + 2y = 102$

$A = xy$

$$x = \frac{102}{3} - \frac{2y}{3}$$

$$x = 34 - \frac{\cancel{2}}{3}\cdot\frac{51}{\cancel{2}}$$

$= 17$

$$A = \left(34 - \frac{2}{3}y\right)y$$

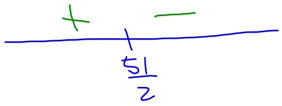
$$A = 34y - \frac{2}{3}y^2$$

$$\frac{dA}{dy} = 34 - \frac{4}{3}y = 0$$

$$\frac{3}{\cancel{4}_{2}}\cdot\overset{17}{\cancel{34}} = \frac{4}{3}y$$

$$y = \frac{51}{2}$$

$x = 17$



$$A = \frac{51}{2}\cdot 17$$

$$= \frac{867}{2}\text{ m}^2$$

1.

$x \cdot y = 192$ $x = \frac{192}{y}$

$S = x + 3y$

$S = \frac{192}{y} + 3y$

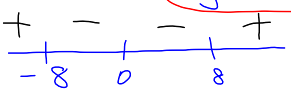
$S' = -\frac{192}{y^2} + 3 = 0$

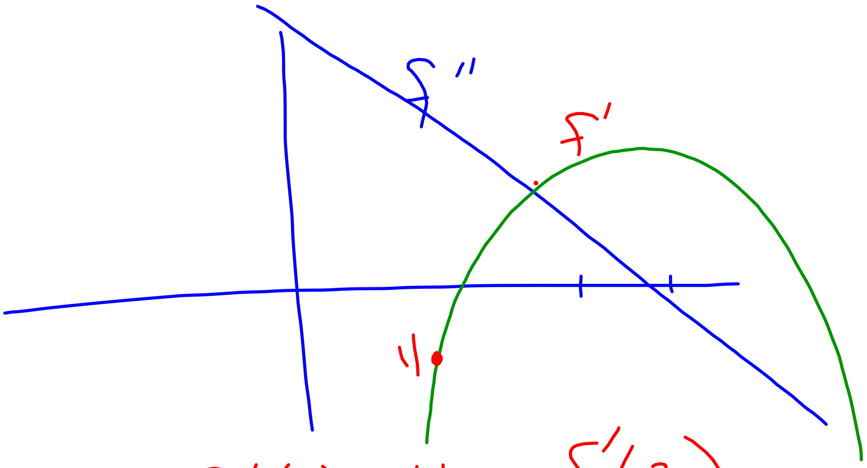
$\frac{192}{y^2} = 3$

$\frac{192}{3} = y^2$

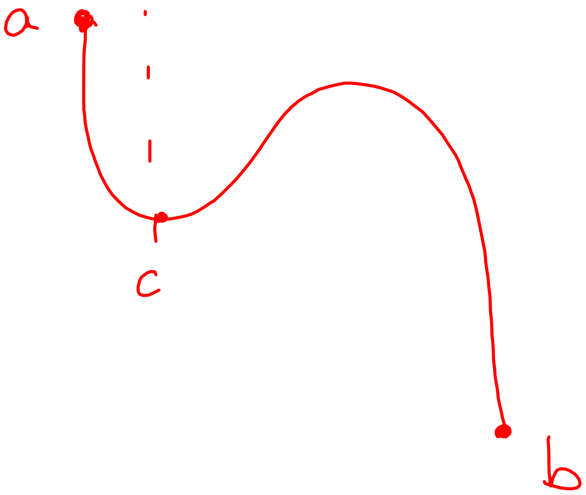
$64 = y^2$

$\boxed{y = 8 \quad x = 24}$





$f'(2) = 11$ $f'(3)$



Series.

Tests:

P $\sum_{n=1}^{\infty} \frac{1}{n^p}$

$p > 1$ con.

$p \leq 1$ div.

$$\begin{tikzcd} a_n \arrow[rr, bend left, "\text{if div.} \quad \text{then div.}"] & < & b_n \arrow[ll, bend left, "\text{if con.} \quad \text{then con.}"] \end{tikzcd}$$

$$\lim_{n \to \infty} \frac{a_n}{b_n} = L$$

Series find x's for which ______ con.

Geom $|r| < 1$

never includes endpts.

Ratio $|\text{ans.}| < 1$

must check endpts.

Error: $\left| f(x) - P(x) \right|$

Taylor
Lagrange

$$|E| < \frac{M(x-a)^{n+1}}{(n+1)!}$$

max value on
the n+1 der
on the given interval

Truncated
Alternating

$$E < \left| \text{nxt term} \right|$$

actual
error

$$\left| f(a) - P(a) \right|$$

Memorized:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdot - - - -$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4} + \cdot - - - -$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdot \cdot \cdot \cdot \cdot$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdot \cdot \cdot -$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3}$$