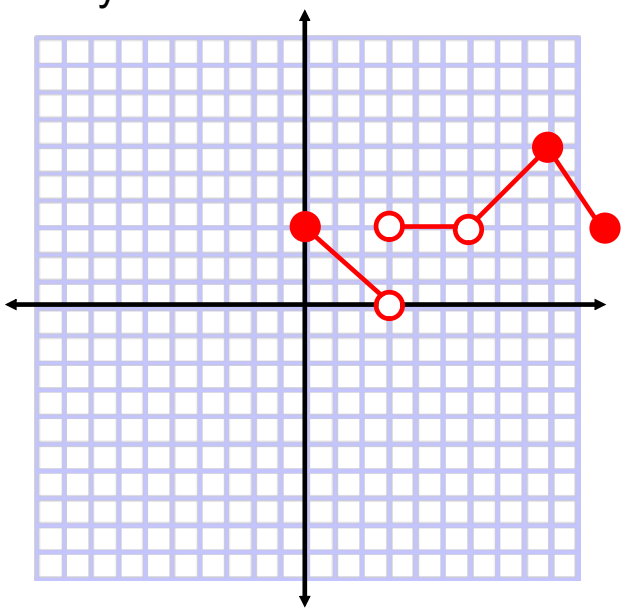
2.3 Continuity



$x = 3, 6$

$(-\infty, 0)$

$(11, \infty)$

Where is the function discontinuous?

Continuous:

$[0,3) \cup (3,6) \cup (6,11]$

9 $\lim_{x\to\infty} \frac{1-\cos x}{x^2} = 0$

45

52 $\lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{\cos x}{x^2} = 0 - 0 = 0$

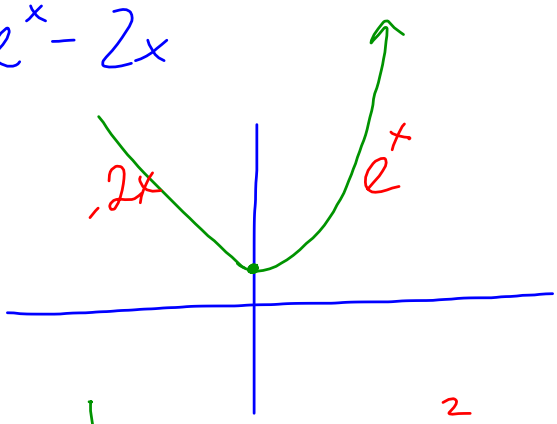
54

55. $-\frac{1}{x^2} \le \frac{\cos x}{x^2} \le \frac{1}{x^2}$

~~65~~

0 0 0

45. $y = e^x - 2x$



$e^{-2} = \frac{1}{e^2}$ $\quad e^2 - 2(2)$

$e^{-10} = \frac{1}{e^{10}}$ $\quad e^{10} - 2(10)$

$e^{-100} = \frac{1}{e^{100}}$ $\quad e^{100} - 2(100)$

52. $f(x) = x \sin\left(\frac{1}{x}\right)$

$f\left(\frac{1}{x}\right) = \frac{1}{x} \sin\left(\frac{1}{x} \cdot \frac{x}{1}\right)$

$\frac{1}{x} \cdot \sin(x) = \frac{\sin x}{x}$

54.

$$f(x) = \begin{cases} \frac{x-2}{x-1} & x \le 0 \\ \frac{1}{x^2} & x > 0 \end{cases}$$

55.

$x \to 1 = 2$

$x \to \infty = -1$

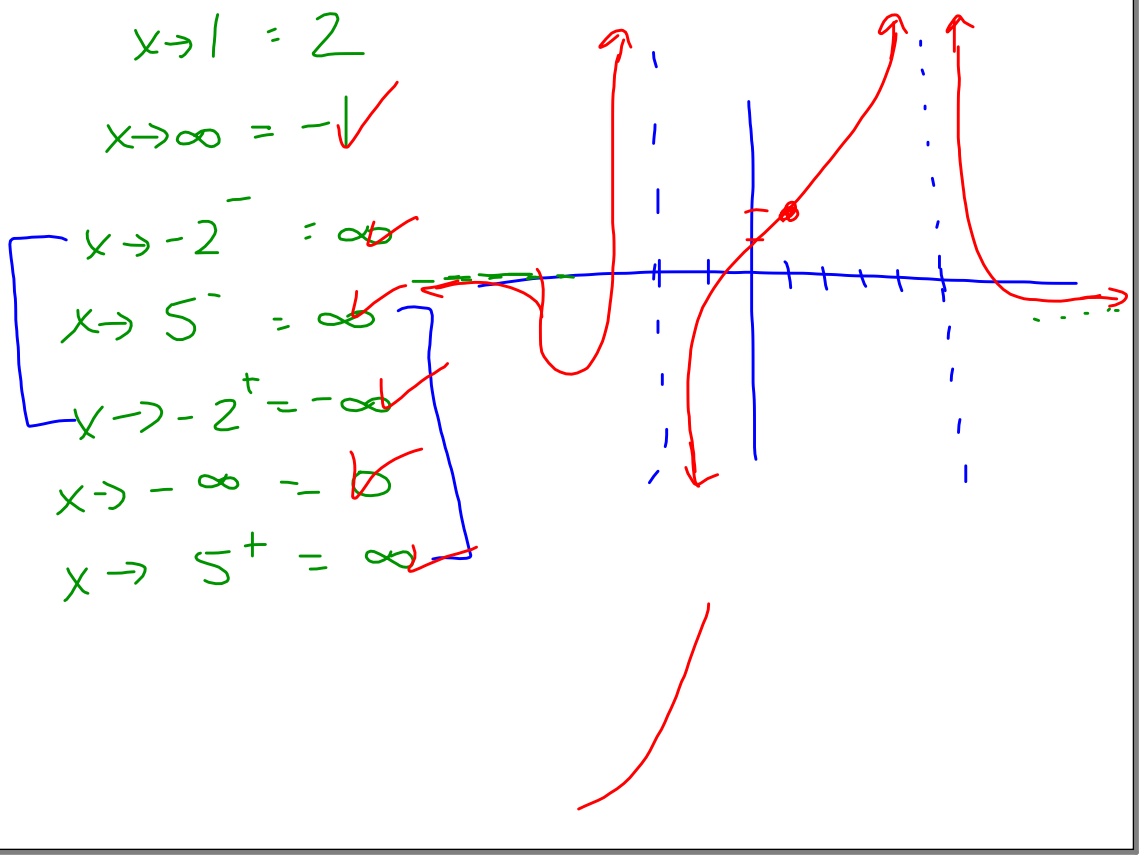
$x \to -2^- = \infty$

$x \to 5^- = \infty$

$x \to -2^+ = -\infty$

$x \to -\infty = 0$

$x \to 5^+ = \infty$



definition of continuity at a point

$f(x)$ is continuous at x = c if

1. $\lim_{x \to c} f(x)$ exists (remember this means left hand = right hand)
2. $f(c)$ exists
3. the answers from #1 = the answers from #2

$$\lim_{x \to c} f(x) = f(c)$$

definition of continuity on an interval

$f(x)$ is continuous on the interval (a,b) if $f(x)$ is continuous at every point in the interval

types of discontinuities

hole (removable discontinuity)

@ x = 0

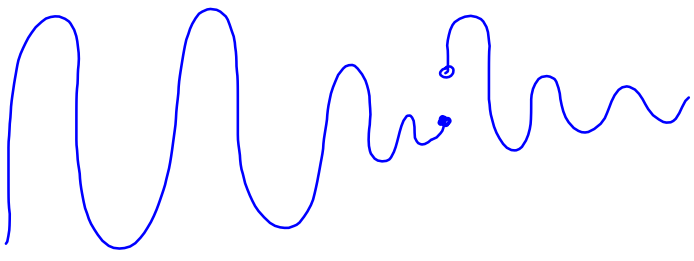
$\frac{x(x+1)}{x(x-3)}$

jump discontinuity

infinite discontinuity

VA @ x = 3

oscillating discontinuity

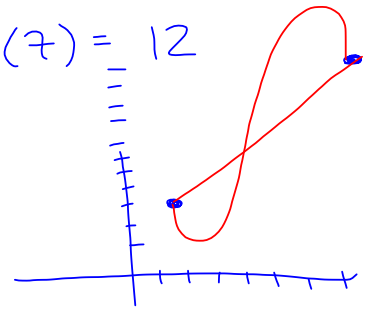


intermediate value theorem for continuous functions

IVT

$f(2) = 3$ $f(x)$ is cont.

$f(7) = 12$



if $f(x)$ cont. on $[a, b]$

and $f(a) < w < f(b)$ then

exists a "c" between a & b

such that $f(c) = w$.