

Definition of a Horizontal Asymptote:

$$\lim_{x \rightarrow \infty} f(x) = b$$

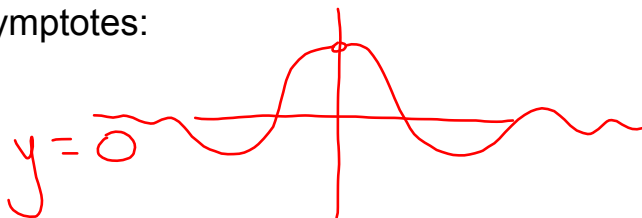
if *or*

then there is a HA at $y = b$

$$\lim_{x \rightarrow -\infty} f(x) = b$$

Find the horizontal asymptotes:

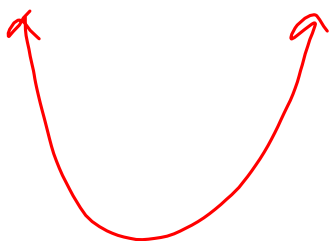
$$f(x) = \frac{\sin x}{x}$$



$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

$$\lim_{x \rightarrow \infty} f(x) = 1$$

$$\lim_{x \rightarrow -\infty} f(x) = -1$$



$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$1. f(x) = \cos\left(\frac{1}{x}\right)$$

$$a. \lim_{x \rightarrow \infty} f(x) = 1$$

$$b. \lim_{x \rightarrow -\infty} f(x) = 1$$

$$c. y = 1$$

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$$f(x) = \frac{4x^3 - 2x + 1}{x - 2}$$

$$e.b.m. = \frac{4x^3}{x} = 4x^2$$

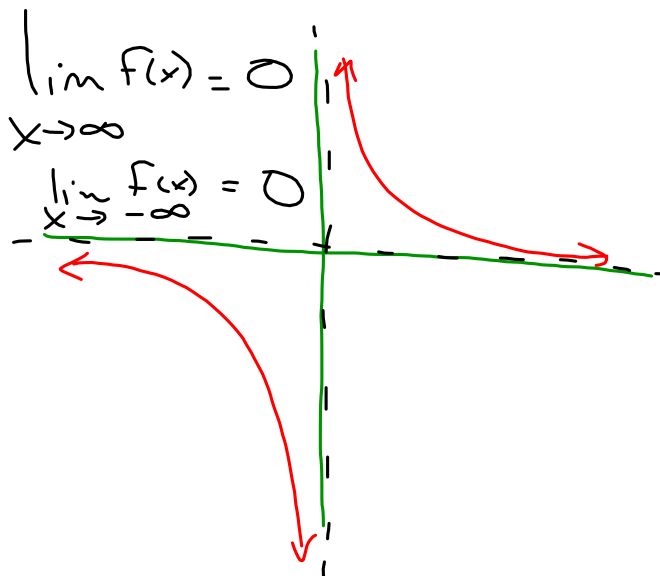
$$x \rightarrow \infty$$

$$x \rightarrow -\infty$$

none

Estimate: (remember this includes both the left and right hand limits!)

$$\lim_{x \rightarrow 0} \frac{1}{x} = \text{DNE}$$



$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right) = \infty$$

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{x}\right) = -\infty$$

Definition of a Vertical Asymptote:

$$\lim_{x \rightarrow a^+} f(x) = \pm\infty$$

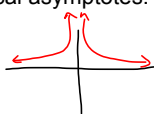
or

then there is a VA at $x = a$

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty$$

Find the vertical asymptotes:

$$f(x) = \frac{1}{x^2}$$



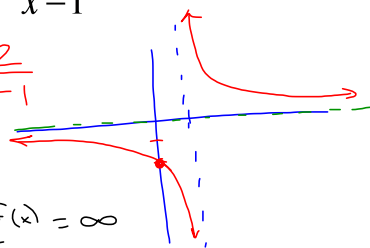
$$x=0 \quad \lim_{x \rightarrow 0^+} f(x) = \infty$$

$$\lim_{x \rightarrow 0^-} f(x) = \infty$$

$$f(x) = \frac{2}{x-1}$$

$$x=1$$

$$0 = \frac{2}{x-1}$$



$$\lim_{x \rightarrow 1^+} f(x) = \infty$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

Find the limits on the left & rt. of the VA:

$$f(x) = \frac{(x+3)^1}{x^2 - 2x - 8} = 0$$

$(x-4)(x+2)$

$\lim_{x \rightarrow -2^+} f(x) = -\infty$ $\lim_{x \rightarrow 4^+} f(x) = \infty$
 $\lim_{x \rightarrow -2^-} f(x) = \infty$ $\lim_{x \rightarrow 4^-} f(x) = -\infty$

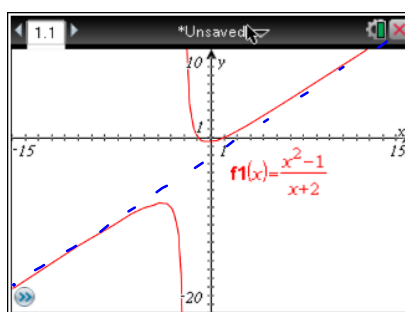
End Behavior Models:

$$f(x) = \frac{x^2 + 1}{x + 2}$$

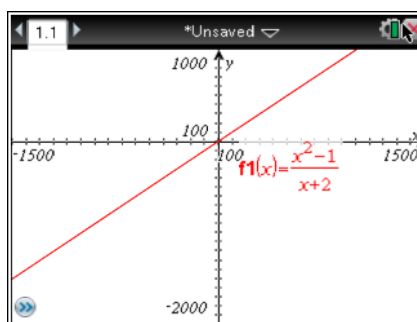
e.b.m. = $\frac{x^2}{x} = x$

Graph $f(x)$ using the following windows:

$[-15, 15]$ $[-20, 10]$
x's *y's*



$[-1500, 1500]$ $[-2000, 1000]$



what is an end-behavior model for $f(x)$:

Find the end behavior models:

$$y = \frac{2x^5 + x^4 - x^2 + 1}{3x^2 - 5x + 7}$$

$$7 - 5x + 3x^2$$

$$e.b.m. = \frac{2}{3}x^3$$

$$\frac{2x^3}{3}$$

$$y = \frac{x+1}{3x^2-4x+5}$$

$$e.b.m. = \frac{1}{3x}$$