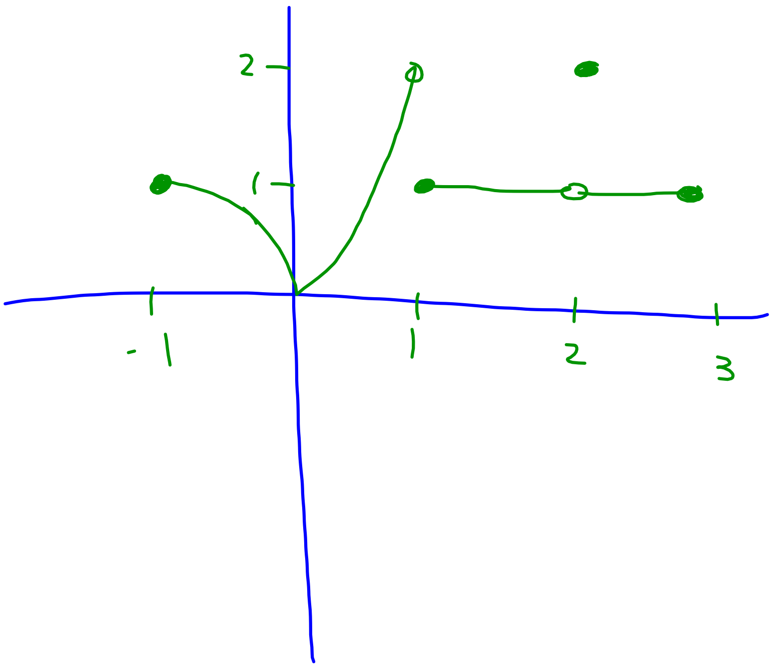


21.

$$\lim_{x \to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$$

$$\lim_{x \to 0} \frac{\cancel{x^2}(5x + 8)}{\cancel{x^2}(3x^2 - 16)} = \frac{8}{-16} = \frac{-1}{2}$$

19.

$$\lim_{x\to 1} \frac{x-1}{x^2-1}$$

$$\lim_{x\to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$$

20. $$\lim_{t\to 2} \frac{t^2-3t+2}{t^2-4}$$

$$\lim_{t\to 2} \frac{(t-1)\cancel{(t-2)}}{\cancel{(t-2)}(t+2)}$$

$$\lim_{t\to 2} \frac{t-1}{t+2} = \frac{1}{4}$$

Sum/ Difference (a) $\lim_{x\to a}[f(x)+h(x)] =$ $\lim_{x\to a} f(x) + \lim_{x\to a} h(x)$

Product (b) $\lim_{x\to a}[f(x)]^2 =$ $\lim_{x\to a} f(x) \cdot \lim_{x\to a} f(x)$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} =$

(d) $\lim_{x\to a}\frac{1}{f(x)} =$

$\lim_{x\to 1} (5x)(x+2)$

Quotient (e) $\lim_{x\to a}\frac{f(x)}{h(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} h(x)}$

(f) $\lim_{x\to a}\frac{g(x)}{f(x)} =$

(g) $\lim_{x\to a}\frac{f(x)}{g(x)} =$

Constant Multiple (h) $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} =$

$2\lim_{x\to a}\frac{f(x)}{h(x)-f(x)}$

$\lim_{x\to 3} 2x = 2\lim_{x\to 3} x$

Given that $\lim_{x\to a} f(x) = -3,\ \lim_{x\to a} g(x) = 0,\ \lim_{x\to a} h(x) = 8,$
find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to a}[f(x)+h(x)] = 5$

(b) $\lim_{x\to a}[f(x)]^2 = 9$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} = 2$

(d) $\lim_{x\to a}\frac{1}{f(x)} = -\frac{1}{3}$

(e) $\lim_{x\to a}\frac{f(x)}{h(x)} = -\frac{3}{8}$

(f) $\lim_{x\to a}\frac{g(x)}{f(x)} = 0$

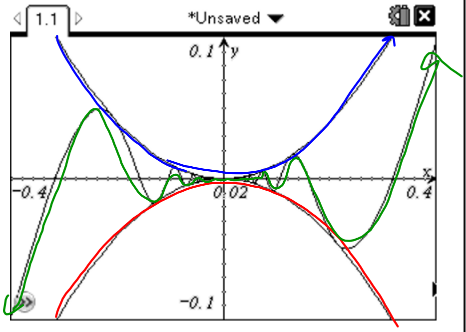
(g) $\lim_{x\to a}\frac{f(x)}{g(x)} =$ DNE

(h) $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} =$ $-\frac{6}{11}$

The Sandwich Theorem

Estimate

$$\lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right)$$



$$-1 \cdot x^2 \le x^2 \cdot \sin\left(\frac{1}{x}\right) \le 1 \cdot x^2$$

$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

$$\lim_{x\to 0}(-x^2) \qquad \lim_{x\to 0} x^2$$

$$0 \quad = \quad 0$$

$$\therefore \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

If $g(x) \le f(x) \le h(x)$ for all x except possibly x=c and

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L \quad \text{then} \quad \lim_{x\to c} f(x) = L$$

If $1 \le f(x) \le x^2 + 2x + 2$ for all x, find $\lim_{x\to -1} f(x)$

$\lim_{x\to -1} 1 \qquad \lim_{x\to -1} (x^2+2x+2)$

$1 = 1$

$\therefore \lim_{x\to -1} f(x) = 1$

If $3x \le f(x) \le x^3 + 2$, evaluate $\lim_{x\to 1} f(x)$.

$\lim_{x\to 0}\left(x^3 \sin x\right)$

$-1 \le \sin x \le 1$

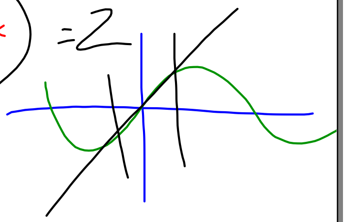
$-x^3 \le x^3 \sin x \le x^3$

$\lim_{x\to 0}(-x^3) \qquad \lim_{x\to 0} x^3$

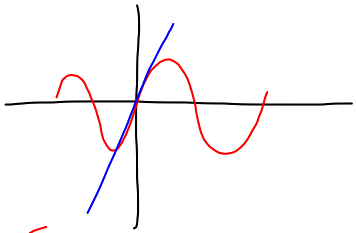
$0 = 0$

$\therefore \lim_{x\to 0} x^3 \sin x = 0$

$$\lim_{x\to 0}\frac{\sin 2x}{x} = 2\left(\frac{\sin 2x}{2x}\right) = 2$$



$$\lim_{x\to 0}\frac{\sin 5x}{2x}$$



$$\frac{5}{2}\left(\frac{\sin 5x}{5x}\right) = \frac{5}{2}$$

$$\lim_{x\to 0}\frac{x+\sin x}{x} = \lim_{x\to 0}\frac{x}{x} + \lim_{x\to 0}\frac{\sin x}{x}$$

$$1+1=2$$