29.

$$r \cdot r = 8r\sin\theta$$

$$r^2 = 8r\sin\theta$$

$$x^2 + y^2 = 8y$$

41.

$$r = 2 - 3\sin\theta$$

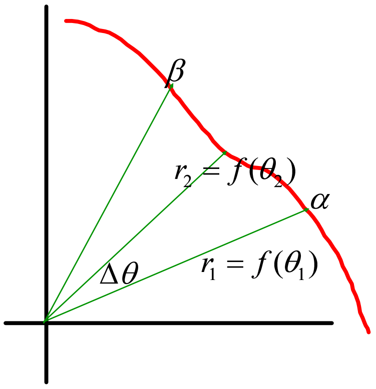
$$x = (2-3\sin\theta)\cos\theta$$

$$y = (2-3\sin\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{(2-3\sin\theta)(\cos\theta) + \sin\theta(-3\cos\theta)}{(2-3\sin\theta)(-\sin\theta) + \cos\theta(-3\cos\theta)}$$

# 10.3b Polar Functions
## Area

Area of a Sector $= \frac{\theta}{2\cancel{\pi}}\cancel{\pi} r^2$ so = $\frac{1}{2}\theta r^2$




Area $\approx \sum_{k=1}^{n} \frac{1}{2}\left(f(\theta_k)\right)^2 \bullet \Delta\theta$

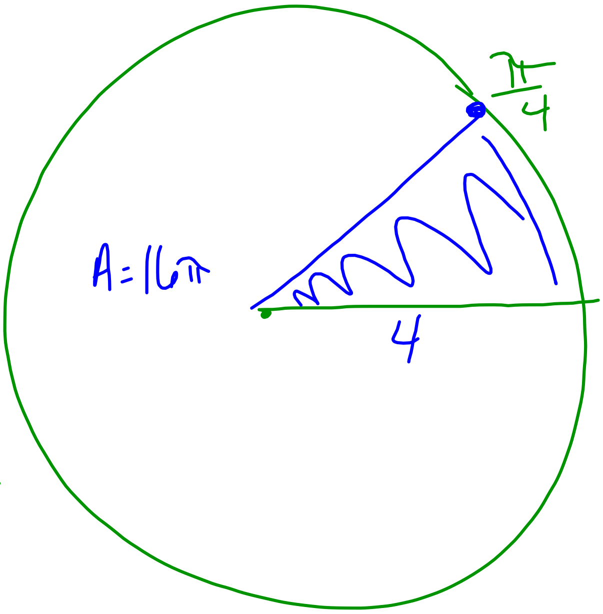
so

Area = $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2}\left(f(\theta_k)\right)^2 \bullet \Delta\theta = \frac{1}{2}\int_{\alpha}^{\beta} \left(f(\theta)\right)^2 d\theta$

$A_{\odot} = \pi r^2$

$\frac{45}{360} \cdot 16\pi$

$\frac{\frac{\pi}{4}}{2\pi} \cdot 16\pi$



Find the area bounded by the graph of $r = 2 + 2\sin\theta$. use Geo file

$$A = \frac{1}{2}\int_0^{2\pi} (2+2\sin\theta)^2 \, d\theta$$

Power Reducer

$$\frac{1}{2}\int_0^{2\pi} \left(4 + 8\sin\theta + 4\underline{\sin^2\theta}\right) d\theta$$

$$\frac{1}{2}\int_0^{2\pi} 4 + 8\sin\theta + \cancel{4}^{2}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right)$$

$$\int_0^{2\pi} \left(2 + 4\sin\theta + 1 - \cos 2\theta\right) d\theta$$

$$\int_0^{2\pi} 3 + 4\sin\theta - \cos 2\theta \quad d\theta$$

$$3\theta - 4\cos\theta - \frac{\sin 2\theta}{2} \Big|_0^{2\pi}$$

$$3(2\pi) - 4(1) - 0 - \left(0 - 4(1) - 0\right)$$

$6\pi$ = area of cardiod

Find the area of one petal of $r = 2\sin 3\theta$.

see 3 petals
really 6 petals

$$0 = 2\sin 3\theta$$

$$\sin 3\theta = 0$$

$$\frac{\cancel{3}\theta}{\cancel{3}} = \frac{-\pi}{3}, \frac{0}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}$$

$$\theta = -\frac{\pi}{3}, \underbrace{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi}, \frac{4\pi}{3}$$

$$\frac{1}{2}\int_0^{\pi/3} (2\sin 3\theta)^2 \, d\theta$$

$$\frac{1}{\cancel{2}}\int_0^{\pi/3} \cancel{4}^{2}\sin^2 \underline{3\theta} \, d\theta$$

$$\int_0^{\pi/3} \cancel{2}\left(\frac{1-\cos 6\theta}{\cancel{2}}\right) d\theta$$

$$\int_0^{\pi/3} 1 - \cos 6\theta \, d\theta$$

$$\theta - \frac{\sin 6\theta}{6}\Big|_0^{\pi/3}$$

$$\frac{\pi}{3} - 0 - (0 - 0)$$

$\frac{\pi}{3}$ area of 1 petal

$\pi$ area of flower

Find the area of one petal of $r = 4\cos 2\theta$.

double this is petal #

to find intervals for a petal

$4\cos 2\theta = 0$

$\cos 2\theta = 0$

$\frac{2\theta}{2} = \frac{1}{2}\left(-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\right)$

$\theta = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots$

$\frac{1}{2}\int_{\pi/4}^{3\pi/4} (4\cos 2\theta)^2 \, d\theta$

$16\cos^2 2\theta \, d\theta$

$\frac{1}{\cancel{2}}\int_{\pi/4}^{3\pi/4} \cancel{16}^{\,4\,\cancel{8}}\left(\frac{1+\cos 4\theta}{\cancel{2}}\right) d\theta$

$4\int_{\pi/4}^{3\pi/4} 1 + \cos 4\theta \, d\theta$

$4\left(\theta + \frac{\sin 4\theta}{4}\right)\Big|_{\pi/4}^{3\pi/4}$

$4\left(\left(\frac{3\pi}{4} + 0\right) - \left(\frac{\pi}{4} + 0\right)\right)$

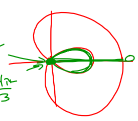
$4 \cdot \frac{\pi}{2} = 2\pi = 1$ petal

$8\pi = 4$ petals

Find the area of the inner loop of: $r = 1 + 2\cos\theta$

$0 = 1 + 2\cos\theta$

$\cos\theta = -\frac{1}{2}$




$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1 + 2\cos\theta)^2 \, d\theta$

$\frac{1}{2}\int_{2\pi/3}^{4\pi/3} \left(1 + 4\cos\theta + \cancel{4}^{\,2}\left(\frac{1+\cos 2\theta}{\cancel{2}}\right)\right) d\theta$

$\frac{1}{2}\left(\theta + 4\sin\theta + 2\theta + \frac{2\sin 2\theta}{2}\right.$

$\frac{1}{2}\left(3\theta + 4\sin\theta + \sin 2\theta\right)\Big|_{2\pi/3}^{4\pi/3}$

$\frac{1}{2}\left(4\pi + 4\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\right) - \left(2\pi + 4\left(\frac{\sqrt{3}}{2}\right) + -\frac{\sqrt{3}}{2}\right.$

$\frac{1}{2}\left(2\pi - \frac{6\sqrt{3}}{2}\right)$

$\pi - \frac{3\sqrt{3}}{2}$