10.2 Vectors

Vector names:

v bold print

$\vec{v}$ $\overline{v}$

$\hat{v}$

#231

component form: shows the components of the vector (Δx and Δy) from standard position

(sometimes called the postion vector)

(x_1, y_1) tail

(x_2, y_2) head

$$\langle (x_2 - x_1), (y_2 - y_1) \rangle$$

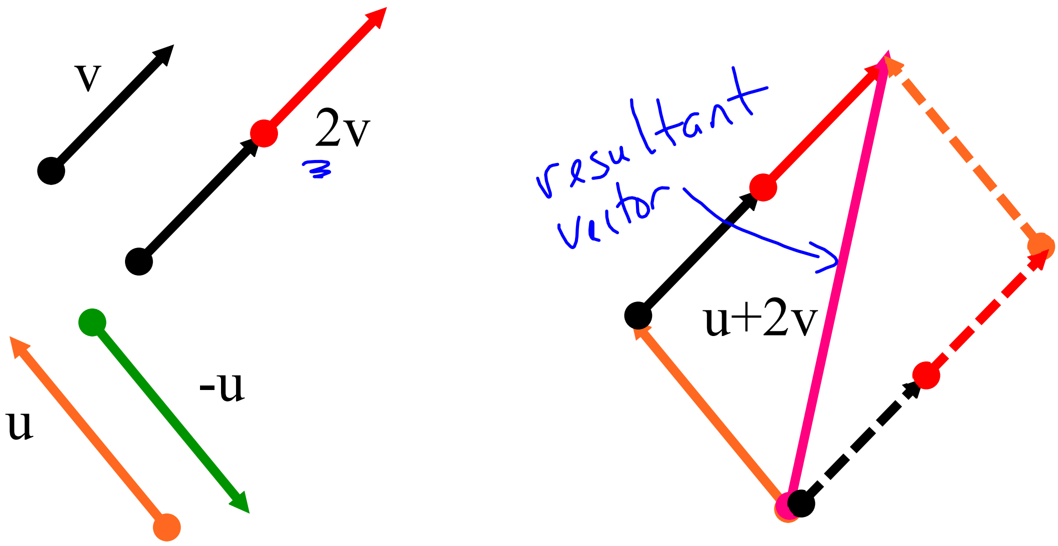
$$\langle \Delta x, \Delta y \rangle$$

Magnitude: length of the vector (use distance formula)

$|u|$ ← magnitude of vector u

Zero vector: has 0 length and no direction

Operations with Vectors #232



vector addition and subtraction: add or subtract the components

scalars: distribute to both the x and y components

Example: $u = \langle 1, 4 \rangle \quad v = \langle -4, 5 \rangle$

$3v = \langle -12, 15 \rangle \quad 2u - 3v = \langle 14, -7 \rangle$

$u + v = \langle -3, 9 \rangle$

#231a

Unit Vector: vector with magnitude of 1

to change to a unit vector in the direction of u: $\dfrac{u}{|u|}$

direction vector

Standard Unit Vectors: $i = \langle 1, 0 \rangle$ $j = \langle 0, 1 \rangle$

all vectors can be written using a linear combination of the standard unit vectors

$v = \langle a, b \rangle$ $v = ai + bj$

$\langle \Delta x, \Delta y \rangle$

direction: θ theta

Find the magnitude and direction of v: $v = \langle 5, 4 \rangle$

$|v| = \sqrt{5^2 + 4^2} = \sqrt{41}$

$\tan\theta = \frac{y}{x}$ $\tan\theta = \frac{4}{5}$

Write a unit vector for v: $\theta = .6747$ rad.

$\frac{v}{|v|} = \left\langle \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}} \right\rangle$ $\sqrt{\left(\frac{5}{\sqrt{41}}\right)^2 + \left(\frac{4}{\sqrt{41}}\right)^2}$

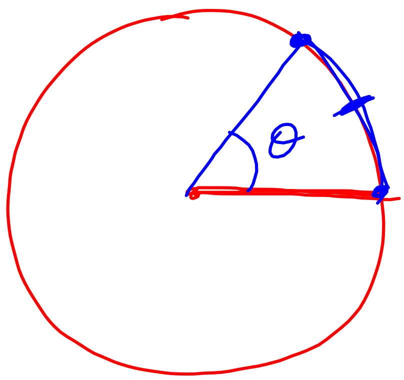
Find the component form of the vector with magnitude 5 and direction 200°.

$v = \langle x, y \rangle$

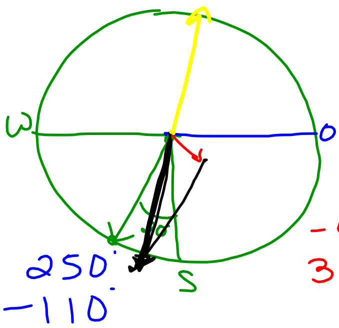
$\cos\theta = \frac{x}{r}$ $\langle r\cos\theta, r\sin\theta \rangle$

$\langle 5\cos 200°, 5\sin 200° \rangle$

$x = r\cos\theta$ $\langle -4.6984, -1.7101 \rangle$



A ship is heading 20° west of south at 15mph. A current is flowing southeast at 3 mph. Find the new speed and direction of the ship.



$S = \langle 15\cos 250°, \ 15 \sin 250° \rangle$

$+ \ C = \langle 3\cos 315°, \ 3 \sin 315° \rangle$

$-45°$

$315°$

$\langle -3.0089, -16.2167 \rangle$

mag. $= 16.4928$

direction $= \tan^{-1}\left(\dfrac{-16.2167}{-3.0089}\right) = 1.38733$

$+ \ \pi$

4.5289

Vectors and Calculus

$r(t) = \langle x(t), y(t) \rangle$ is the position vector at any time t.

$v(t) = \langle x'(t), y'(t) \rangle$ is the velocity vector at any time t.

$a(t) = \langle x''(t), y''(t) \rangle$ is the acceleration vector at any time t.

speed is: $|v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ or the magnitude of the velocity vector

direction vector: $\frac{u}{|u|}$ or $\left\langle \frac{x}{|u|}, \frac{y}{|u|} \right\rangle$ or $\frac{x}{|u|} i + \frac{y}{|u|} j$

distance traveled: $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

final position: $\left(x_o + \int_a^b \frac{dx}{dt}, y_o + \int_a^b \frac{dy}{dt} \right)$

Given the position vector, find the velocity vector, acceleration vector, speed, and the direction vector.

$r(t) = \langle 2t^3 + 1, 4t^2 \rangle$

Direction Vector

$\left\langle \frac{2t^3+1}{\sqrt{(2t^3+1)^2 + (4t^2)^2}}, \frac{4t^2}{\sqrt{(2t^3+1)^2 + (4t^2)^2}} \right\rangle$

$v(t) = \langle 6t^2, 8t \rangle$

$a(t) = \langle 12t, 8 \rangle$

Speed: $\sqrt{(6t^2)^2 + (8t)^2}$

A particle moves in the xy-plane so that at any time t, the position of the particle is given by

$$x(t) = 2t^3 - 15t^2 + 36t + 5,\ y(t) = t^3 - 3t^2 + 1, \text{ where } t \geq 0.$$

For what value(s) of t is the particle at rest?

$K = \langle 2t^3 - 15t^2 + 36t + 5,\ t^3 - 3t^2 + 1 \rangle$

$v(t) = \langle 6t^2 - 30t + 36,\ 3t^2 - 6t \rangle$

$6t^2 - 30t + 36 = 0$

$6(t^2 - 5t + 6) = 0$

$6(t-2)(t-3) = 0$

$t = 2, 3$

$3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0, 2$

$t = 2$ because $v(t) = 0$ for both x & y

A particle moves in the xy-plane in such a way that its velocity vector is $\langle 3t^2 - 4t,\ 8t^3 + 5 \rangle$.

If the position vector at $t = 0$ is $\langle 7, -4 \rangle$, find the position of the particle at $t = 1$. Find the distance the particle moves from $t = 0$ to 3.

Distance: $\int_0^3 \sqrt{(3t^2 - 4t)^2 + (8t^3 + 5)^2}\, dt$

Position: $7 + \int_0^1 (3t^2 - 4t)\,dt,\ -4 + \int_0^1 (8t^3 + 5)\,dt$

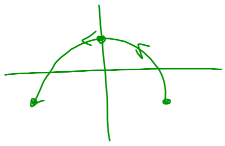
$7 + \left(t^3 - 2t^2 \Big|_0^1 \right);\ -4 + \left(2t^4 + 5t \Big|_0^1 \right)$

$7 - 1 \qquad -4 + 7$

$(6,\ 3)$

$x = 2\sin t \qquad y = \cos 2t \qquad 1.5 \le t \le 4.5$

t	1.5	2.5	3.5	4.5
x				
y				

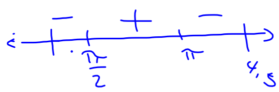


$$-\frac{\cancel{2}\sin 2t}{\cancel{2}\cos t}$$

$$\frac{dy}{dx} = \frac{-\sin 2t}{\cos t} = 0$$

$$\sin 2t = 0$$

$$\frac{\cancel{2}t}{\cancel{2}} = \frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}$$



$t = 0, \frac{\pi}{2}, \pi$